Spectral Line Calibration Techniques with Single Dish Telescopes

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A Quick Review
The Rayleigh-Jeans Approximation

- Planck Law for Blackbody radiation:
  \[ B = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT} - 1} \]
- If \( \nu \sim \text{GHz} \), often \( h\nu \ll kT \). Taylor series gives:
  \[ B = \frac{2kT^2}{\lambda^2} \]
- Source flux in Rayleigh Jeans limit:
  \[ S = \frac{2kT}{\lambda^2} \int \Omega S T(\theta,\phi) d\Omega \]
- If brightness temperature is constant across source:
  \[ S = \frac{2kT}{\lambda^2} \Omega_S \]

A Quick Review

Antenna Temperature

- Antenna theorem:
  \[ A_e \Omega_A = \varepsilon_r \lambda^2 \]
- Measured flux:
  \[ S = \frac{2kT_A}{\lambda^2} \Omega_A = \frac{2kT_A}{A_e} \]
- Temperature:
  \[ T_A = (A_e/\lambda^2) \int \int T_{\text{src}}(\theta,\phi) P_n(\theta,\phi) d\Omega = (\varepsilon_r/\Omega_A) \int \int T_{\text{src}}(\theta,\phi) P_n(\theta,\phi) d\Omega \]
Antenna Temperature

• Temperature:
  \[
  T_A = \left(\frac{A_e}{\lambda^2}\right) \int \int T_{src}(\theta,\phi) P_n(\theta,\phi) d\Omega \\
  = \left(\frac{\varepsilon}{\Omega_A}\right) \int \int T_{src}(\theta,\phi) P_n(\theta,\phi) d\Omega
  \]

• Point Source:
  \[
  T_A = \left(\frac{\varepsilon}{\Omega_A}\right) \int \int T_{src}(\theta,\phi) d\Omega = \varepsilon_r \left(\frac{\Omega_S}{\Omega_A}\right) T_{avg}
  \]

• Source > Beam, \( T = T_{const} \)
  \[
  T_A = \left(\frac{\varepsilon_r T_{const}}{\Omega_A}\right) \int \int P_n(\theta,\phi) d\Omega = \varepsilon_r \left(\frac{\Omega_{const}/\Omega_A}{\Omega_b}\right) T_{avg}
  \]

\( \varepsilon_r = \) fractional power of antenna transmission; \( \Omega_A = \) antenna solid angle; \( P_{nh} = \) antenna power pattern; \( \Omega_b = \) solid angle subtended by main beam and side lobes

Minimal Detectable Temperature

Set by the system noise
\[
T_{sys} = T_A + \frac{1}{\varepsilon} T_R + \frac{T_{LP}[1/\varepsilon - 1]}{\sqrt{\Delta \nu t n}}
\]

Sensitivity is rms noise of system:
\[
\Delta T_{rms} = K_S T_{sys} / \sqrt{\Delta \nu t n}
\]
\[
\Delta B_{rms} = \left(\frac{2k}{\lambda^2}\right) K_S T_{sys} / \sqrt{\Delta \nu t n}
\]
\[
\Delta S_{rms} = \left(\frac{2k}{A_e}\right) K_S T_{sys} / \sqrt{\Delta \nu t n}
\]

\( T_{min} \sim 5 \times \Delta T_{rms} \)

\( T_R = \) receiver temperature; \( T_{LP} = \) transmission line temperature; \( \varepsilon = \) efficiency transmission; \( K_S = \) telescope sensitivity constant \((\sim 1)\);
\( \Delta \nu = \) pre-detection bandwidth (Hz); \( t = \) integration time, one record; \( n = \) number of records
A Quick Review

Antenna Temperature

- Telescope observes a point source (flux density $S$)
- Telescope feed replaced with matched load (resistor)
- Load temperature adjusted until power received equals power of the source
- This is equal to the Antenna Temperature

Determining the Source Temperature
\[ T_{\text{meas}}(\alpha, \delta, az, za) = T_{\text{src}}(\alpha, \delta, az, za) + T_{\text{RX}} + T_{\text{gr}}(za, az) + T_{\text{cel}}(\alpha, \delta, t) + T_{\text{CMB}} + T_{\text{atm}}(za) \]

\[ T_{\text{meas}} = T_{\text{source}} + T_{\text{everything else}} \]
Determining $T_{\text{source}}$

Relative Intensities

ON - OFF

$(T_{\text{source}} + T_{\text{everything else}}) - (T_{\text{everything else}})$

channel

Arbitrary Units

Determining $T_{\text{source}}$

Relative Intensities

$(\text{ON} - \text{OFF})/\text{OFF}$

$\left[ (T_{\text{source}} + T_{\text{everything else}}) - (T_{\text{everything else}}) \right] / T_{\text{everything else}}$

channel

channel

channel
Choosing the Best Off

\[ T_{\text{source}} = \frac{(\text{ON} - \text{OFF})}{\text{OFF}} \]

Baseline Fitting with Best Fit Line

Image on right courtesy of C. Conselice
Baseline Fitting with Best Fit Line

- Simplest & most efficient method

- Not feasible if:
  - Line of interest is large compared with bandpass
  - Standing waves in data
  - Cannot readily fit bandpass

- Errors are primarily from quality of fit

Frequency Switching

Raw spectra
Choosing the Best Off

Frequency Switching

• Allows for rapid switch between ON & OFF observations
• Does not require motion of telescope
• Can be very efficient

Disadvantages:
• Frequency of line of interest must be known
• System must be stable
• Will not work with time or frequency varying baselines
Position Switching

- Little a priori information needed
- Typically gives very good results
- Disadvantages:
  - System must be stable in time
  - Requires re-pointing the telescope
  - Results in time off source
  - Sky position must be carefully chosen
  - Source must not be too extended

- Best results if the same sky (AZ, EL) position used
Beam Switching

• Same idea as position switching
• Removes need to move telescope

• Disadvantages/Caveats:
  Requires hardware to exist
  Sky position must be carefully chosen
  Source must not be extended beyond throw

Beam Switching – 2 Beams

• Same idea as position switching
• Removes need to move telescope
• Always on source!

• Disadvantages/Caveats:
  Requires additional hardware
  Sky position must be carefully chosen
  Source must not be extended beyond beam separation
Baseline Fitting with an Average Fit

Alternative if frequency switching is not an option
May lose detailed information for individual fits
System must be very stable

Position Switching on Strong Continuum
Possibly only alternative if $T_{\text{src}} > \text{few} \times T_{\text{sys}}$

Designed to remove residual standing waves

Result:

$$R = \frac{[\text{On}(\nu) - \text{Off}(\nu)]_{\text{source}_1}}{[\text{On}(\nu) - \text{Off}(\nu)]_{\text{source}_2}}$$

Choosing the Best Off

Position Switching on Strong Continuum

From ATOM 2001-02 by Ghosh & Salter
Choosing the Best Off

Position Switching on Strong Continuum

Determining $T_{\text{source}}$

\[
\frac{(\text{ON} - \text{OFF})/\text{OFF}}{(T_{\text{source}} + T_{\text{everything else}}) - (T_{\text{everything else}})} / T_{\text{everything else}}
\]

Result \[= \frac{T_{\text{source}}}{T_{\text{system}}}
\]

Units are: % System Temperature

Need to determine system temperature to calibrate data
Determining System Temperature

\[ T_{\text{meas}}(\alpha, \delta, \alpha, \delta, \alpha, \delta) = T_{\text{src}}(\alpha, \delta, \alpha, \delta) + T_{\text{RX}} + T_{\text{gr}}(\alpha, \delta, \alpha, \delta) + T_{\text{cel}}(\alpha, \delta, t) + T_{\text{CMB}} + T_{\text{atm}}(\alpha, \delta, \alpha, \delta) \]

\[ T_{\text{meas}} = T_{\text{source}} + T_{\text{system}} \]
Theory

Measure various components of $T_{sys}$:

- $T_{RX}$ — Can be readily measured/monitored
- $T_{CMB}$ — Well known (2.7 K)
- $T_{cel}(\alpha,\delta,t)$ — Can be determined from other (tel.) measurements
- $T_{atm}(za)$ — Can be determined from other (tel.) measurements
- $T_{gr}(za,az)$ — Can be calculated

1 - Noise Diodes
Determining System Temperature

1 - Noise Diodes

\[
\frac{T_{\text{src}}}{T_{\text{sys}}} = \frac{\text{ON} - \text{OFF}}{\text{OFF}}
\]

\[
\frac{T_{\text{diode}}}{T_{\text{sys}}} = \frac{\text{On} - \text{Off}}{\text{Off}}
\]

\[
T_{\text{sys}} = T_{\text{diode}} \cdot \frac{\text{Off}}{(\text{On} - \text{Off})}
\]

1 - Noise Diode Measurement Considerations

- Frequency dependence

Lab measurements of the GBT L-Band calibration diode, taken from work of M. Stennes & T. Dunbrack - February 14, 2002
1 - Noise Diode Measurement Considerations

- Time stability

\[
\sigma^2_{\text{measured value}} = \sigma^2_{\text{standard cal}} + \sigma^2_{\text{instrumental error}} + \sigma^2_{\text{loss uncertainties}}
\]

Typically measured against another diode or other calibrator

Errors inherent in instruments used to measure both diodes

Measurements often done in lab. Have numerous losses through path from diode injection to back ends
1 - Noise Diode Measurement Considerations

- Frequency dependence
- Time stability
- Accuracy of measurements

\[ \sigma^2_{\text{measured value}} = \sigma^2_{\text{standard cal}} + \sigma^2_{\text{instrumental error}} + \sigma^2_{\text{loss uncertainties}} \]

\[ \sigma^2_{\text{total}} = \sigma^2_{\text{freq. dependence}} + \sigma^2_{\text{stability}} + \sigma^2_{\text{measured value}} + \sigma^2_{\text{conversion error}} \]

Determining System Temperature

The Y-Factor (Two Diodes)

\[ Y = \frac{T_1 + T_{\text{off}}}{T_2 + T_{\text{off}}} \]

\[ T_{\text{off}} = \frac{T_1 - YT_2}{Y - 1} \]

- Can be more accurate than just one diode
- Ignores effects of the antenna
2 - Hot & Cold Loads

- Same idea as two diodes
- Takes antenna into account
- True temperature measurement (no conversion)
Determining System Temperature

2 - Hot & Cold Loads

\[ T_{\text{off}} = \frac{T_1 - YT_2}{Y - 1} \]
Determining System Temperature

2 - Hot & Cold Loads

Requires a reliable load able to encompass the receiver, with response fast enough for on-the-fly measurements

Same idea as two diodes
• Takes antenna into account
• True temperature measurement (no conversions)

Requires a reliable load able to encompass the receiver, with response fast enough for on-the-fly measurements
3 - Astronomical Measurements

- Use sources with well determined fluxes for calibration
- Easy to obtain high spectral frequency resolution
- Uses same hardware as observations

Requires extremely reliable measurements of source flux

Error will always be dominated by source error

Determining System Temperature

Determining $T_{sys}$

Theory:
- Needs detailed understanding of telescope & structure
- Atmosphere & ground scatter must be stable and understood

Noise Diodes:
- Can be fired rapidly to monitor temperature
- Requires no 'lost' time
- Depends on accurate measurements of diodes

Hot/Cold Loads:
- Can be very accurate
- Observations not possible when load on
- Must be in mm range for on-the-fly measurements

Astronomical Measurements:
- Can be very accurate
- Uses the same hardware as astronomical measurements
- Must know source fluxes extremely well
Determining $T_{source}$

$$T_{source} = \frac{(ON - OFF)}{OFF} T_{system}$$

Blank Sky or other

From diodes, Hot/Cold loads, etc.

Telescope response has not been accounted for!

Determining Telescope Response
Telescope Response

**Telescope Response**

- Main Beam Brightness:
  \[ T_{MB} = \eta_{\text{beam}} T_{\text{measured}} \]

- Flux Density:
  \[ S = \frac{2k}{\lambda^2} \int \int T(\theta,\phi) P_n(\theta,\phi) d\Omega \]

Units: W m\(^{-2}\) Hz\(^{-1}\) or Jy (1 Jy = 10\(^{-22}\) W m\(^{-2}\) Hz\(^{-1}\))

\( \Omega \) = solid angle of tel. pattern; \( \eta_{\text{beam}} \) = telescope efficiency; \( \lambda \) = wavelength; \( k \) = constants, \( T \) = temperature; \( P \) = antenna power pattern

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**1 - Ideal Telescope**

Accurate gain, telescope response can be modeled

Can be used to determine the flux density of ‘standard’ continuum sources

Not practical in cases where telescope is non-ideal
(blocked aperture, cabling/electronics losses, ground reflection, etc)
1 - Ideal Telescope

Telescope Response

2 - ‘Bootstrapping’

Observe source with pre-determined fluxes
Determine telescope gain

\[
\frac{T_{\text{source}} \cdot \frac{1}{\text{GAIN}} - \frac{\text{ON}}{\text{OFF}}}{\text{OFF}} \quad \frac{T_{\text{system}}}{\text{GAIN}}
\]

\[
\text{GAIN} = \frac{\frac{\text{OFF}}{\left( \frac{\text{ON}}{\text{OFF}} \right)} \cdot T_{\text{system}}}{T_{\text{source}}}
\]
2 - ‘Bootstrapping’

- Useful when gain is not readily modeled
- Offers ready means for determining telescope gain

- Requires flux of calibrator sources be known in advance
- Not practical if gain changes rapidly with position

3 - Pre-determined Gain Values

Pre-determined Gain curves:

- Allows for accurate representation of gain at all positions
- Saves observing time
- Can be only practical solution
3 - Pre-determined Gain Values

Telescope Response

Average Gain [(polA+polB)/2]:

\[
\text{gainavg}(\text{az,za}, f=1415\text{MHz}) = 10.999 - 0.10291\times 10^{-08}\cos(\text{az}) - 1.3225\times 10^{-08}\sin(\text{az}) + 1.1642\times 10^{-08}\cos(2\times \text{az}) - 7.3761\times 10^{-07}\sin(2\times \text{az}) - 0.20990\cos(3\times \text{az}) - 0.098026\sin(3\times \text{az})
\]

\[
\text{gainavg}(\text{az,za}, f=1175\text{MHz}) = 11.378 - 0.081304\times 10^{-08}\cos(\text{az}) - 0.026763\times 10^{-08}\sin(\text{az}) - 1.0319\times 10^{-08}\cos(2\times \text{az}) - 3.1292\times 10^{-07}\sin(2\times \text{az}) - 0.17180\cos(3\times \text{az}) - 0.046071\sin(3\times \text{az})
\]

\[
\text{gainavg}(\text{az,za}, f=1300\text{MHz}) = 11.265 - 0.095145\times 10^{-08}\cos(\text{az}) - 0.004248\times 10^{-08}\sin(\text{az}) - 0.2271\times 10^{-07}\cos(2\times \text{az}) - 9.0897\times 10^{-07}\sin(2\times \text{az}) - 0.26895\cos(3\times \text{az}) - 0.006216\sin(3\times \text{az})
\]

\[
\text{gainavg}(\text{az,za}, f=1375\text{MHz}) = 11.114 - 0.10412\times 10^{-08}\cos(\text{az}) - 0.023915\times 10^{-08}\sin(\text{az}) - 0.0094938\times 10^{-08}\cos(2\times \text{az}) - 8.1956\times 10^{-07}\sin(2\times \text{az}) - 0.22135\cos(3\times \text{az}) - 0.0074295\sin(3\times \text{az})
\]

\[
\text{gainavg}(\text{az,za}, f=1550\text{MHz}) = 10.786 - 0.10748\times 10^{-08}\cos(\text{az}) - 0.019365\times 10^{-08}\sin(\text{az}) - 0.0075530\times 10^{-08}\cos(2\times \text{az}) - 7.8976\times 10^{-07}\sin(2\times \text{az}) - 0.20972\cos(3\times \text{az}) - 0.14330\sin(3\times \text{az})
\]
3 - Pre-determined Gain Values

Pre-determined Gain values:

- Allows for accurate representation of gain at all positions
- Saves observing time
- Can be only practical solution

Caveat:
Observers should always check the predicted gain during observations against a number of calibrators!

Telescope Response

\[
T_{\text{source}} = \frac{(\text{ON} - \text{OFF})}{\text{OFF}} T_{\text{system}} \frac{1}{\text{GAIN}}
\]

Blank Sky or other

From diodes, Hot/Cold loads, etc.

Theoretical, or Observational

Great, you’re done?
A Few Other Issues

Pointing

OFFSET ~50% BEAMWIDTH

GAIN 100%

GAIN 40%

Results in reduction of telescope gain
Typically can be corrected in telescope pointing model or offset
Focus

Results in reduction of telescope gain
Can be corrected mechanically if rcvr/subreflector can be adjusted

Other Issues

Side Lobes*

• Allows in extraneous or unexpected radiation

• Can result in false detections, over-estimates of flux, incorrect gain determination

• Solution is to fully understand shape and variance in side lobes
Other Issues

Comatic Error

- Sub-reflector shifted perpendicular from main beam
- Results in an offset between the beam and sky pointing

Astigmatism

- Deformities in the reflectors

Can result in false detections, over-estimates of flux, incorrect gain determination
Solution is to fully understand beam shape
List of useful references pp 310-311 in book