RADIO TELESCOPES and MEASUREMENTS at RADIO WAVELENGTHS

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RADIO TELESCOPES

Distinguishing Features:

1. Collect only a single mode of the electromagnetic field

2. Angular resolution limited by diffraction

3. Coherent devices used for detection, so are sensitive to phase of the incident signal

Note: Distinguishing feature #3 becomes blurred at submillimeter wavelengths where incoherent detectors which directly measure incident power (e.g. bolometers) are used.
Analysis:

This is an electromagnetic field problem

Components are:
- Antenna
- Feed System
- Receiver

Normal radio astronomy operation is in the “receive” mode

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Analyze using

The RECIPROCITY THEOREM

The relative sensitivity of the antenna + feed to signals coming from different directions is the same as the relative angular distribution of power radiated if you replace the receiver by a transmitter.

We carry out analysis in “transmit mode” in which power from feed system illuminates the antenna, which in turn radiates into different directions.
The MODES of the electromagnetic field

- are the configurations of E and H which satisfy Maxwell’s equation
- depend on geometry of conductors and dielectrics
  **MODES and an object’s dimensions**
- An object with dimensions \( \gg \lambda \) has many spatial modes
- As its dimensions are reduced \( \rightarrow \lambda \), you get finite set of modes. You can arrange a situation where only one mode can propagate \( \Rightarrow \)

**SINGLE MODE SYSTEM**

\[ \lambda_0 = \text{Wavelength in free space} \]

\[ 2a < \lambda_0 \]

\[ a < \lambda_0 < 2a \]

\[ \lambda_0 \leq a \text{ More than one mode (Multimode propagation)} \]

WITH SINGLE MODE, YOU KNOW EXACT CONFIGURATION OF EM FIELD — CAN DESIGN COUPLING TO DEVICES, OTHER TRANSMISSION LINES, …

- Single mode waveguide
- Field horn (multimode)
- Single coaxial cable
- Transmitters in amplifier
NOTE ON POLARIZATION:

- SINGLE NODE SYSTEM HAS ONLY ONE WELL-DEFINED POLARIZATION STATE
- IN SOME CASES MAY BE "STANDARD", C.G. LINEARLY POLARIZED
- DESIRED POLARIZATION CAN BE ACHIEVED BY COMBINING SIGNALS FROM 2 SINGLE MODE TRANSMISSION LINES WITH APPROPRIATE AMPLITUDE AND PHASE

THE FEED SYSTEM

- Couples radiation from the antenna to the receiver
- Is itself fundamentally an antenna system, but has one input being a SINGLE MODE TRANSMISSION LINE and the other input being FREE SPACE
- Radiation pattern of feed horn determines illumination of the antenna
A TYPICAL FEED SYSTEM

(Think reciprocally here !)

- Takes field configuration in single mode transmission line (waveguide)
- Gradually expands transverse dimensions to the desired aperture size
- Lets field radiate from aperture to illuminate the main reflector (or whatever antenna is being used)

Key points to remember:

1. Radiation from feed system is itself determined by diffraction from feed.

2. The critical issue is to illuminate the antenna as you wish.

3. Many feeds have radiation pattern which is fairly close to GAUSSIAN form. This is very convenient.

ANTENNA IS A PHASE TRANSFORMER
Aperture Plane Field Distribution

\[ E_{ap} (x,y) = |E_p(x,y)| e^{i \phi_p(x,y)} \]

contains all information!

If you are interested in far field

\[ D \gg \frac{2D_e}{\lambda} \]

then you can just do a Fourier transform

\[ E_{ff} (\theta, \phi) = \mathcal{F} \left( E_{ap} (x,y) \right) \]

Electric Field Distribution in far-field
is just (proportional to) Fourier Transform
of Aperture Field Distribution.

Because of diffraction, the far-field pattern will not be unidirectional (plane wave)
Different amounts will go in different directions

Remember reciprocity theorem – this means as a receiver you will be sensitive not
great to signal coming from single direction
but you have a distribution of sensitivity
to radiation incident from different
directions. Usually given in terms of polar graph

Normalized Power Pattern

\[ P_n (\theta, \phi) = \frac{P(\theta, \phi)}{P(0,0)} \]

\[ P(\theta, \phi) = |E_{ff} (\theta, \phi)|^2 \]
\( P(0,0) \) is taken as direction bounds which give you maximum power or maximum sensitivity (called boresight direction by antenna engineers).

Normalized Power Pattern is Sensitivity Relative to that on boresight.

\[ P_n(\theta) \]

\[ \theta \rightarrow \]

\[ \theta = \text{angle off boresight} \]

\[ \phi = \text{polar angle} \]

Note: Often, it's negative as well as positive.

Even though not strictly for planar system, should do in correct 3D plan plot.

For convenience use Decibels

\[ P_n(\text{dB}) = 10 \log_{10} [P_n] \]

<table>
<thead>
<tr>
<th>dB</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>-10</td>
<td>0.1</td>
</tr>
<tr>
<td>-13</td>
<td>0.05</td>
</tr>
<tr>
<td>-17</td>
<td>0.02</td>
</tr>
<tr>
<td>-20</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Sidelobe levels typically -10 to -20 dB.

Lower is better especially for making maps of complex sources (confusion) and getting very good calibrations (CBR).
Polar diagram of normalized power pattern of uniformly illuminated aperture

GAUSSIAN ILLUMINATION
UNBLOCKED APERTURE

\[ U = \pi Da \sin \theta / \lambda \]

\[ \Gamma_u = 30 \text{ dB} \]

\[ \Gamma_u = 0 \text{ dB} \]

\[ \Gamma_u = 5 \text{ dB} \]

\[ \Gamma_u = 10 \text{ dB} \]

\[ \Gamma_u = 15 \text{ dB} \]

\[ \Gamma_u = 20 \text{ dB} \]

\[ \Gamma_u = 25 \text{ dB} \]

\[ \Gamma_u = 30 \text{ dB} \]
**Fourier Transform Relationship Review**

- Large Edge Taper => Small Exp at edge
  => little truncation by edge
  Result in Far Field:
  - few side lobes
  - large beam width (of main lobe)

- Small Edge Taper
  => Large Exp at edge
  => significant truncation by edge
  Result in Far Field:
  - high side lobes
  - narrow main lobe

Need to choose for applications!

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**Beam Width**

Beam width is not trivial issue, but common usage is to quote angular distance between points where $P_n = 0.5$

$$\Delta \Theta_{FWHM} = 45^\circ$$ for example

The main lobe normalized power pattern typically is fairly Gaussian $\Rightarrow$
from FWHM you can estimate beam width to $-3\text{dB}$, $-20\text{dB}$, etc. but not good below this level since you have very poor attenuation
Preceding discussion of Fourier transform relationship is general.

For circular unblocked antenna with Gaussian illumination having edge taper $T_e(\text{dB})$

$$\Delta \theta_{\text{FWHM}} \approx \left[ 1.02 + 0.0135 T_e(\text{dB}) \right] \frac{\lambda}{D}$$

Max sidelobe rel. $\leq -17 \text{dB} - T_e(\text{dB})$

Effective area + aperture efficiency

Relevant for observations of point sources

Assume polarization state of antenna matched to that of incident wave (left polarized) or for antennas - assume source unpolarized.

Then, whatever pol. that antenna is sensitive to, you get half of what could be received from unpolarized source.

$$P_{\text{collected}} = \frac{4}{3} A_e \times S_v \times S_v$$

Effective area of antenna

bandwidth

(Definitive)

 Coil

\(A\)}
Effective Area = \frac{2 \cdot \text{Power Collected}}{S_0 \cdot B_0}

Example:
- source flux density: \( S_0 = 10^{-6} \text{W/m}^2 \cdot \text{Hz}^{-1} \)
- bandwidth: \( B_0 = \frac{1}{2} \text{Hz} \) (10^5 Hz)
- if you collect 10^{-6} W power:
  \[ A_c = \frac{2 \times 10^{-16}}{10^{10} \text{W/m}^2 \cdot \text{Hz}^{-1} \cdot 10^5 \text{Hz}} = 2 \times 10^4 \text{ m}^2 \]

Aperture Efficiency
\[ \varepsilon_A = \frac{A_c}{A_p} = \frac{\text{Effective Area}}{\text{Physical Area}} \]

Measures "collection efficiency" when observing a plane wave.

Theoretical:
\[ \varepsilon_A = \text{efficiency of coupling between aperture plane and plane wave} \]

\( A_p = \text{aperture plane field distribution} \)
\( E_{pw} = \text{plane wave field distribution} \)

For \( E_p = \text{gaussian} \):
\[ E_p(r) = \exp \left( \frac{-r^2}{2r_0^2} \right) \]
\( r < r_0 \)
\( E_{pw} = \text{uniform over aperture} \]
\( \varepsilon_{pw} = 0.8865, \quad 0 > r_0 \)

\[ \varepsilon_A = \left( \frac{\langle E_p \rangle \langle E_{pw} \rangle^2}{\langle E_p \rangle^2 \langle E_{pw} \rangle} \right)^2 \]

where \( \langle xy \rangle = \int xy \, dA \)

\[ \varepsilon_A = \left[ \frac{\int_0^{r_0} \exp \left( -\frac{r^2}{2r_0^2} \right) r^2 \, dr}{\int_0^{r_0} \exp \left( -\frac{r^2}{2r_0^2} \right) \, dr} \right]^2 \]

\[ = \frac{2r_0}{\exp \left( -\frac{r_0^2}{2r_0^2} \right)} \left( 1 - e^{-\frac{r_0^2}{2r_0^2}} \right)^2 \]

\[ \varepsilon_A = \frac{2r_0}{\exp \left( -\frac{r_0^2}{2r_0^2} \right)} \left( 1 - e^{-\alpha} \right)^2 \]

where \( \alpha = \text{illumination parameter} \)
\[ \left( \frac{R_{\text{pw}}}{R} \right)^2 = \left( \frac{D}{2r_0} \right)^2 \]
The aperture efficiency can be considered to be product of two factors:

1. Spillver efficiency = fraction of power interrupted by aperture
   \[ \varepsilon_s = 1 - e^{-2\alpha} \]

2. Taper efficiency = how close illumination is to ideal uniform distribution
   \[ \varepsilon_t = \frac{1}{\alpha} \frac{(1 - e^{-\alpha})^2}{1 - e^{-2\alpha}} \]

\[ \varepsilon_A = \varepsilon_s \cdot \varepsilon_t \]
Treatment of aperture efficiency can be extended to include:

- Blockage (secondary support legs)
- Defocusing (phase error)

Consider antenna as phase transformer:

Spherical wave from feed

Plane wave in aperture plane

Requires correct phase unification.

\[ R = f \Rightarrow \phi'(f) + \phi(f) = 0 \]

- Plane wave output
- Still like spherical wave
- Phase variation in aperture plane
- Broader beam
Blackbody Radiation and Antenna Temperature

Suppose you connect a single mode, one-dimensional transmission line to our blackbody. Assume Rayleigh Jeans limit (k \nu c / k T \approx 1). In bandwidth $\delta \nu$

$$P = \delta \nu (kT S \lambda)$$

[Does not depend on any angles or area]
\[ P = kT \sigma \nu \]

\[ \text{\( T = 77 \, K \)} \]
\[ \text{\( \nu = 10^6 \, \text{Hz} \)} \]
\[ \text{\( k = 1.38 \times 10^{-23} \, \text{W/K} \)} \]
\[ P = 1 \times 10^{-15} \, \text{W} \]

Antenna is receiving power \( P_{\text{rec}} \) from its enclosure.

\( P_{\text{rec}} = P_{\text{rad}} \to \text{blackbody enclosure} \)

\( P_{\text{rec}} \) = Power flowing down transmission line connecting \( T \) to absorbing load

\( P_{\text{rec}} = kT \sigma \nu \)

In equilibrium: \( T = T_a \)

\[ P_{\text{rec}} = kT_a \sigma \nu \]
\[ T_h = \frac{T_{rec}}{4\pi \nu} \]

Defines the Antenna Temperature in terms of power received by antenna in any situation.

Equal to temp. of ideal absorbing sheet that would radiate that power (in RTFD).

Example - for a point source (ampl.)

\[ P_{rec} = \frac{1}{2} A e S_v \delta \nu \]

Which we now see produces antenna temperature

\[ T_h = \frac{P_{rec}}{4\pi \nu} = \frac{A e S_v \delta \nu}{2\pi} \]

Antenna Temperature is convenient way to express power collected by antenna

Obviating any kind of source.

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**ANTENNAS & EXTENDED SOURCES**

**Antenna Performance**

- point sources - aperture efficiency - effective area
- extended sources - normalized power pattern \( P_n(\theta, \phi) \)

**Definition**

Antenna Solid Angle

\[ \Omega_h = \iint_{\text{solid angle}} P_n(\theta, \phi) \, d\Omega \]

**Theorem** - For any single mode radiating system

\[ A_c \Omega_h = \lambda^2 \]

Use example: if you have given antenna has beamwidth at - you intuitively feel that response is less directional. This also means that effective area (on axis) is reduced to compensate for increase in \( \Omega_h \)
In general sources do not fill entire aperture pattern. You can write for a uniform source:

$$T_A = \frac{\int (\int_{-\infty}^{\infty} P_{\theta}(\theta) d\theta) dA}{\int_{-\infty}^{\infty} p_{\theta}(\theta) dA}$$

The term in brackets is the CONTINUING EFFICIENCY for source of that angular size.

For symmetric antenna pattern, adopt circular source of angular radius $\Theta_0$,

$$E(\Theta_0) = \frac{\int_{0}^{\Theta_0} P_{\theta}\sin\Theta' d\Theta'}{\int_{0}^{\infty} P_{\theta}\sin\Theta' d\Theta'}$$

If $\Theta_0$ just $\approx$ size of main lobe, $E(\Theta_0)$ is called the MAIN BEAM EFFICIENCY

$$T_{MB} = \frac{T_A}{E_{\text{MB}}}$$

This gives brightness temp. Strictly speaking, only for unit, source of radius $\Theta_0$, not often used.

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**Graph**

- **Label**: 
  - $E_{MB} \to 1.0$ as $\alpha$ increases (EDGE TAPERED ILLUMINATION)
  - WHAT IS THE PRICE?
  - $E_A$ (goes down)
  - $\Delta B_{\text{peak}}$ (increases)
  - LOW ATTENUATION EFFICIENCY

**Axes**
- $\alpha = \text{(APERTURE RADIUS/GAUSSIAN BEAM RADIUS)}$
- **Values**
  - $E_{MB}$
  - $E_A$
  - $\Delta B_{\text{peak}}$
**Reflectors + Surface Errors**

general idea applies to any system of reflectors.

Surface errors produce phase errors in far field region of aperture.

Result is deviation of aperture plane phase distribution

![Diagram](attachment:image.png)

We assume that amplitude distribution is unaffected.

Coupling to plane wave is maximized if aperture field has uniform phase front.

**Aperture Field Coupling to Plane Wave**

\[ |\langle \mathbf{E}_a | \mathbf{E}_{p0} \rangle |^2 = \left| \int \mathbb{E}(\mathbf{y}) e^{i \mathbf{k} \cdot \mathbf{r}} \, d\mathbf{y} \right|^2 \]

\[ \approx \sum_a |\mathbb{E}(\mathbf{y}_a)|^2 \]

These errors reduce coupling to plane wave and hence the aperture efficiency.

\[ \frac{\mathbf{E}_a}{\mathbf{E}_a(0)} = \frac{\left| \int \mathbb{E}(\mathbf{y}) e^{i \mathbf{k} \cdot \mathbf{r}} \, d\mathbf{y} \right|^2}{\left| \int \mathbb{E}(\mathbf{y}) \, d\mathbf{y} \right|^2} \]

For small field errors, \( e^{-\alpha (\psi - \psi_0)} = 1 - \frac{1}{2} \alpha (\psi - \psi_0)^2 \ldots \)
Surface errors in small Sp limit:

put in series expansion but set to zero all terms like \( \int \exp(\lambda') \phi(\lambda') \lambda' d\lambda' \) because average value can be arbitrarily adjusted.

Keep only leading term, which gives:

\[
\frac{E_\alpha}{E_\alpha(0)} = 1 - \frac{\int \int E_\alpha(\lambda') \phi^2(\lambda') \lambda' d\lambda' d\lambda''}{\int E_\alpha(\lambda') \lambda' d\lambda'}
\]

\[
= 1 - \frac{\int \phi_\text{rms}^2}{\lambda}
\]

\[
= 1 - \left( \frac{4\pi \phi_\text{rms}}{\lambda} \right)^2
\]

value \( \phi_\text{rms} \) in \( \text{rms} \) SURFACE.

For reduction in \( E_\alpha \) by 1 dB (20\%) you must have \( \phi_\text{rms} = 0.5 \) median.

This corresponds to \( \phi_\text{rms} = \frac{\lambda}{2\pi} \).

More sophisticated treatment

Ruze, Proc. IEE vol. 57, pp 685-678 April 1966

Assumptions

1. Many regions contribute

2. Random distribution with amplitude given \( \phi_\text{rms} \)

\[ \text{Arrival (\pm D)} \]

Any specific set of arrows results in

1. Loss in gain
2. Sensitivity to radiation from other directions than beam.

Ruze treated statistically and found
SURE ANALYSIS OF SURFACE ERRORS

\[ \epsilon_a \text{ on axis reduced but the reduction in } \epsilon_a \text{ on axis means energy must go somewhere else.} \]

Errors move sensitivity from on-axis direction to range of directions defined by ERROR PATTERN.

\[ \text{ON AXIS: } \epsilon_a = \epsilon_a^{\text{(perfect)}} - \left( \frac{\text{RMS Error}}{\lambda} \right)^2 \]

\[ \text{ERROR PATTERN: ANGULAR SIZE } \propto \frac{\lambda}{\rho} \quad \rho = \text{CORRELATION LENGTH OF SPOKE} \]

\[ \epsilon_{\text{RMS}} = \text{Root mean sq. of surface errors} = (\text{combine all reflectors separately}) \]

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Fig. 13. Comparison of measured and predicted patterns, HAYSTACK (15.745 Gc/s).

**SURE RESULT:**

\[ \epsilon_{\text{RMS}} = \frac{\lambda}{16} \]

**GIVES** \( \epsilon_a = 0.5 \text{ DF} \)

**VALUE FOR SAME ANTENNA WITH NO ERRORS**
ARECIBO 305m ANTENNA
APERTURE PHASE ERRORS BEFORE ALIGNMENT
\( \sigma_{\text{rms}} \approx 13 \text{ mm} \)

ARECIBO ANTENNA
APERTURE PLANE PHASE ERRORS
AFTER 1st ITERATION OF ALIGNMENT
\( \sigma_{\text{rms}} \approx 5 \text{ mm} \)
Telescope Optics

Radio telescopes = Phase Transformers
Special Lenses = Beam wave or aperture plane

Single Reflector = Paraboloid
- Axis (symmetric)
- Offset

Dual Reflector
Cassegrain = Paraboloid + Hyperboloid
Gregorian = Paraboloid + Ellipsoid

- Symmetric Cassgrain
CSC: 85 m
JCMT: 15 m

- Symmetric Gregorian
Effelsberg: 100 m

- Offset Gregorian
BTL: 5 m

- Offset Gregorian
ATE 80 m
100 m
110 m
Telescope Configurations

- Symmetric Cassegrain
  - Backstop from secondary and support rays
- Offset Cassegrain

Geometrical optics forms focal point (λ/2)
Also provides desired phase transformation
For radio arrays (larger λ)

Allen Telescope Array (ATA)
6m dia. offset

[Diagram of a telescope array with labeled parts]
GBT 10cm dia off-axis

Blockage

Not issue for GBT space-borne antennas for the work.

High efficiency antennas (HFA)
BLOCKAGE - THINK IN TERMS OF FOURIER TRANSFORM OF APERTURE PLANE FIELD

\[ \mathcal{E}_{\text{eff}} \propto \mathcal{F} \left( \mathcal{E}_{\text{ap}} \right) \]

\[ p_n(\theta, \phi) \propto |\mathcal{E}_{\text{eff}}|^2 \]
14 m FCRAO Antenna - Subreflector only
FAR FIELD POWER PATTERN

14 m FCRAO Antenna Aperture Illumination
(Subreflector Blockage Only)
14 m PERSO ANTENNA
SUBREFLECTOR + SUPPORT LEGS

14 m PERSO ANTENNA
FAR FIELD POWER PATTERN
CONCLUSIONS

While it is obviously important for design engineers to understand antennas and feed systems, it is also important that radio astronomers who want to get the most from their single dish antenna appreciate these issues.

Antennas are phase transformers coupling free space to single mode transmission lines.

You need to think of antennas as diffraction-limited, single mode, single–polarization systems.

The antenna illumination pattern, which is typically close to a Gaussian, determines performance parameters.
These include
- efficiency
- beam width
- side lobe levels

You should be aware of the insidious effects of spillover, near- and far-sidelobes, and feed system scattering.

All of these affect the interpretation of your data!

A further issue in the short-wavelength portion of any antenna’s operating range is surface errors, which

- reduce aperture & beam efficiency
- increase sidelobe levels
- broaden the main beam