How to listen to what exotic Pulsars are telling us!

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...in this talk

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Pulsar timing

- How regularly do pulsars spin?
- Can we learn something from tracking their rotations?

Every single pulse from a radio pulsar has a unique shape! However, when we add many such pulses, we obtain a unique shape – the pulsar’s *pulse profile*. The intensity and shape of this are frequency dependent.

This is the fundamental principle behind pulsar timing!
Pulsar timing

- For an isolated pulsar, the main parameters measured by timing are:
  1. *dispersion measure* (DM)
  2. *position in the sky*
  3. *rotational period*
  4. *first derivative of rotation period.*
- From these parameters alone, one can learn a lot about a given pulsar!

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Pulsar timing

- The group velocity of a radio wave in a plasma is slightly smaller than $c$.
- Therefore, after traveling for many centuries through space, pulsar signals arrive at the Earth at slightly different times: the lower the frequency, the larger the delay. These delays are of the order of ms or s.
Pulsar timing

- The delay at a given frequency is given by:

\[ f(F) = 4.15 \, \text{DM} \left( \frac{F}{\text{MHz}} \right)^{-2} \, (\text{ms}) \]

Where

\[ \text{DM} = \int_0^L n_e(\ell) \, d\ell \, (\text{cm}^{-3} \, \text{pc}) \]

- It is because of this effect that we need spectrometers to observe pulsars!
- We can measure the amount of plasma between the Earth and the pulsar!
- The first stage of pulsar data processing is called de-dispersion, where we correct for these dispersive delays.

Pulsar timing

- There are two main techniques for removal of the effect of interstellar dispersion:
  1. Incoherent de-dispersion: The signal is detected at a variety of spectral channels, and then time delays are applied to each channel according to the dispersion relation. Advantage: simple and effective. Caveat: there is still some signal dispersion within each channel.
  2. Coherent de-dispersion: the signal is processed before detection. Pulse recovery is perfect. Caveat: computationally intensive / bandwidth limited.
Pulsar timing

- The Earth’s (and telescope’s!) motion will cause an annual effect on the pulse times of arrival (TOAs) at the telescope location.
- To correct this effect, we must know the pulsar’s precise position in the sky!
- In practice, the opposite is done: the position in the sky is determined from the annual effect on the TOAs, i.e., the Earth’s motion is used to determine the pulsar’s sky position.
- This technique has allowed the determination of some sky positions to precisions of a few micro-arcseconds!

Pulsar timing

- The pulsar’s period can be measured with astounding precision:
- In a 1-hour observation, a 3.6-ms pulsar will rotate 1,000,000 times. An error in the rotational period will, after one hour, produce a shift in the TOAs, (relative to prediction with wrong period) that is 1,000,000 times larger.
- If one could measure the arrival times of the pulses to a precision of 1 µs, then one can determine the 3.6-ms rotational period to a precision of $10^{-12}$s (9 decimal places).
Pulsar timing

- If phase connection can be maintained (i.e., if we can count the pulses unambiguously), then the precision in the measurement of the rotational period increases linearly with time.
- Therefore, an error in the period of $10^{-12}$ s after one hour translates into an error of $10^{-17}$ s after ten years.
- This uncertainty will be further divided by the square root of the number of rotational phase measurements.
- This means that some pulsar periods can be determined to 14 significant decimal places or more!

Pulsar timing

- However, rotational periods are not constant. For every single pulsar in the Galactic disk, the pulse period is observed to increase with time, i.e., $dP/dt > 0$.
- This increase is normally linear with time. This produces a quadratic increase in the delay or the rotational pulse phase relative to a phase prediction with no.
- Therefore, the uncertainty of $dP/dt$ is proportional to $1/T^2$.
- For most pulsars, this effect is relatively small. Measurements have to be made for more than one year to decouple the annual (nearly sinusoidal) effect of the Earth’s motion on the TOAs from the quadratic effect of $dP/dt$ on the TOAs.
Pulsar timing

- Increase in $P$ results from loss of rotational energy:
  \[
  \dot{E} = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = I \Omega \dot{\Omega}
  \]

- Energy is radiated mostly as dipole radiation:
  \[
  \dot{E} = \frac{2}{3c^2} |m|^2 \Omega^4 \sin^2 \alpha
  \]

- Evolution of spin frequency:
  \[
  \dot{\Omega} = - \text{const} \Omega^n = - \left( \frac{2m^2 \sin^2 \alpha}{3c^3 I} \right) \Omega^3
  \]

Pulsar timing

- The pulsar’s age $\tau$ can be estimated, integrating the spindown and assuming that $P \gg P_0$:
  \[
  \tau = \frac{1}{n - 1} \frac{P}{P} = \frac{P}{2P} \text{ (if } n = 3)\]

- Assumptions ($n = 3$, $P \gg P_0$) may be wrong, but $\tau$ is normally a good estimator of the age.

- For the Crab pulsar, real age is 949 years (in 2003), and $\tau$ is 1237 years!
Pulsar timing

- The pulsar’s magnetic field can also be estimated:

\[
B = \frac{|m|}{R^3} = \frac{3c^2 I}{8\pi^2 \alpha R^6 \sin^2 \alpha} \frac{P \dot{P}}{P^2} = 3.2 \times 10^{15} \sqrt{\frac{P \dot{P}}{P^2}} \text{(Tesla)}
\]

- We assumed: \( R = 10 \text{ km}, I = 2/5 \ M R^2 = 10^{38} \text{ kg m}^2, \alpha = 90 \text{ degrees} \).
- Most pulsars have fields of \( 10^8 \text{ T} \), i.e., tens of millions of times more powerful than anything that can be produced on Earth.

Pulsar timing

- Most pulsars show significant departures from simple, uniformly slowing rotation. The two main departures are:
  - **Glitches.** This corresponds to starquakes, caused by a sudden change in the configuration of the magnetic field (for most pulsars, \( B \) seems to increase after each glitch).
  - **Timing noise.** The origin of this phenomenon is unknown!
- The younger the pulsar, and the more intense its magnetic field, the larger are the torques applied to it surface. The phenomena listed above seem to be related to these torques...
Pulsar timing

- The Crab pulsar is a particularly noisy one. For this reason, it has been monitored daily for more than 30 years. By 1999, it had already glitched 13 times.

Pulsar evolution

- Plotting the periods and the period derivatives, we can see that radio pulsars divide mainly in two groups:
  1. *Canonical pulsars*, with periods from 33 ms to 8.5 s and period derivatives larger than $10^{-17}$ s/s.
  2. *Recycled pulsars*, with periods from 1.5 to ~200 ms and small period derivatives.
Pulsar evolution

- Recycled pulsars have much smaller magnetic fields and, (generally) smaller periods than canonical pulsars.
- They tend to be much older: while the ages for canonical pulsars are of the order to 1 kyr to 100 Myr, the typical age of a recycled pulsar is a few Gyr.
- A much larger percentage of recycled pulsars have binary companions.
- Recycled pulsars are divided in two main classes:
  1. Fast-spinning pulsars in circular orbits with low-mass white dwarf companions
  2. Slower pulsars in eccentric orbits with other neutron stars
  3. For each of these two classes there are corresponding isolated objects in the same location in the P-Pdot diagram.

Pulsar evolution

- Millisecond Pulsars with white dwarf companions are formed in Low-Mass X-ray Binaries (LMXBs), where mass is being transferred from a low-mass star into a neutron star. This accretion process lasts hundreds of millions of years, and produces very fast rotation.
- Double neutron stars result from High Mass X-Ray Binaries (HMXBs), where mass is being transferred from a massive star into a neutron star. This stage lasts tens of millions of years, and therefore recycling is not so pronounced.
- The formation of isolated, fast millisecond pulsars is still not understood!
Pulsar evolution

- A distinctive feature of the formation model for MSP/white dwarf systems is the prediction that the orbits should be circular.
- The order of magnitude of the eccentricity of these orbits has been predicted theoretically, and it is in excellent agreement with observation.

The rotation of most recycled pulsars is perfect, i.e., it can be described, to within observational accuracy, as a perfect, featureless slowdown.

- Some recycled pulsars have better long-term stability than atomic clocks!
- Uses are manifold... And have absorbed much time and effort over the last 23 years, with many spectacular results:
  1. Tests of General Relativity using binary pulsars
  2. Constraints of the equation of state for cold, superdense matter
  3. Search for gravitational waves produced by giant black hole mergers.
  4. Probe newtonian systems, like planetary systems and globular clusters.
Tests of gravitational theories using binary pulsars

- In 1974, Russel Hulse and Joe Taylor discovered PSR B1913+16, in the constellation Aquila (the Eagle), during a systematic 430-MHz survey of the Galactic plane at Arecibo.

- It has a DM of 168.77 cm$^{-3}$ pc and a rotational period of 59 ms. This was the second shortest period known at the time.
- This was the first binary pulsar to be discovered.
Tests of gravitation...

- The pulsar and the companion star are orbiting the common center of mass. The orbital period is 7h45m and the eccentricity is 0.61.
- This is an *astrophysically clean system* – no companion is detected, implying that the companion is degenerate. We have two point masses with motion influenced solely by their mutual gravitation.

Orbit of a Binary Pulsar

Tests of gravitation...

- When timing pulsars (or measuring radial velocities with the aid of spectral lines) one has access to the line-of-sight velocity, which causes a *Doppler shift* on the wavelength of the spectral line or a modulation on the period of the pulsar.
- True rotational period not known – radial velocity can not be measured!
Tests of gravitation...

- In the case of pulsar timing, having a clock in the system allows us to measure the range relative to the center of mass of the binary, normally with a precision of the order of 1 km or less!
- This makes pulsar timing thousands of times more precise for measuring orbital parameters.
- This feature is unique to pulsars, and is the fundamental reason why they are superior astrophysical tools.

Tests of gravitation...

- Five Keplerian parameters can normally be measured from fitting the velocity (or, preferably, the delay) curves: orbital period ($P_b$), projected size of the orbit, in light seconds ($x$), eccentricity ($e$), longitude of periastron ($\omega$) and time of passage through periastron ($T_0$). A non-changing Keplerian orbit is exactly what is predicted by Newtonian gravity.
- Without access to information on transverse velocities, the individual masses of the components ($m_1$ and $m_2$) and the inclination of the system ($i$) cannot be measured, but...
Tests of gravitation...

- The mass function, a relation between these three quantities, can be measured to excellent precision, as it depends on two observable parameters:

\[
f(m_1, m_2, i)/M_\odot \equiv \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}
= x^3 \left( \frac{2\pi^2}{P_b} \right) \left( \frac{1}{T_\odot} \right)
\]

\[
T_\odot \equiv \frac{GM_\odot}{c^3} = 4.925490947 \text{ ms}
\]

Tests of gravitation...

- For most binary pulsars, and spectroscopic binaries, there is no information on the transverse velocities, so this is as far as we can go.
- However, the PSR B1913+16 system is so extreme, and pulsar timing is so precise, that some effects of general relativity became detectable!
- Three such effects have been measured in this system:
  1. Periastron advance (\(\dot{\omega}\)).
  2. Rotational slowdown due to gravitational redshift and Lorenz time dilation (Einstein delay, \(\gamma\)).
  3. Orbital decay due to emission of gravitational waves (\(dP_\odot/dt\)).
Tests of gravitation...

- Relativistic perihelion advance had been observed for the orbits of several planets in the solar system, most notably Mercury.
- The periastron of PSR B1913+16 advances 4.226607(7) degrees/year. The daily periastron advance is the same as Mercury’s perihelion advance in a century...

Tests of gravitation...

- The Einstein delay was also measured: \( \gamma = 0.004294(1) \) s. The pulsar slows down visibly when it is near the companion, accumulating a delay relative to the prediction of constant rotational period. General relativity predicts that time itself slows down in an intense gravitational field (in this case, that of the companion object).
- The pulsar also rotates more slowly due to special-relativistic time dilation – it is traveling faster near the companion object.
These two effects determine the mass and inclination of the system! This happens because, according to general relativity, they depend on the known Keplerian parameters and the masses of the two objects:

\[ M = m_1 + m_2, \quad n_b = \frac{2\pi}{P_b} \]

\[ \dot{\omega} = 3n_b^{5/3}(MT_\odot)^{2/3}(1 - e^2)^{-1} \]

\[ \gamma = n_b^{-1/3}em_2(2m_2 + m_1)M^{-4/3}T_\odot^{2/3} \]

The masses of the individual components (and, from the mass function, the inclination of the system!) are only well determined if we assume that General relativity is right.

This is the most precise measurement of any mass outside the solar system!
Tests of gravitation...

- A third relativistic effect is measurable: The orbital period is becoming shorter.
- General relativity predicts this to be due to the loss of energy caused by emission of gravitational waves. This depends only on quantities that are already (supposedly) known:

\[
\dot{P}_b = -\frac{192}{5} n_b^{5/3} f_e m_1 m_2 M^{-1/3} T_\odot^{5/3}
\]

\[
f_e = \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) (1 - e^2)^{-7/2}
\]

- Prediction: the orbital period should decrease \(-2.40247 \times 10^{-12}\) s/s (or 75 μs per year!)

Tests of gravitation...

- Value observed (after subtraction of the Galactic acceleration of the binary) is:
  \(-2.4085(52) \times 10^{-12}\) s/s. The agreement is perfect!
- **GENERAL RELATIVITY GIVES A SELF-CONSISTENT ESTIMATE FOR THE MASSES OF THE TWO COMPONENTS OF THE BINARY!**
- **GRAVITATIONAL WAVES EXIST!**
Tests of gravitation...

- Since then, other systems have been discovered where one can do more than one test of General Relativity:
- For PSR B1534+12, we have a total of 3 tests of general relativity.
- This happens because for this system we can detect the Shapiro delay, which allows two independent measurements of the companion mass and inclination of the system.

Tests of gravitation...

- For the recently discovered Double Pulsar (PSR J0737–3039A and B), we have a total of 4 tests of general relativity – the mass ratio is the constraint unavailable for any previous pulsar!
- Tests of gravitation with this system will be by far the most precise ever.
Probing the equation of state for super-dense matter

- The exact dimensions of neutron stars are not known – this quantity is almost impossible to observe.
- This happens because at the center of a neutron star, the density is of the order of 1 billion tons cm\(^{-3}\), i.e., tens to hundreds of times larger than in atomic nuclei. There is no unique theoretical model for the behavior of matter at such high densities.
- For a given mass, the size and moment of inertia of a neutron star are determined by the relation between pressure and density for degenerate cold matter – the thermal pressure is many orders of magnitude smaller than the degenerate Fermi pressure. This relation is known as the equation of state (EOS).

...super-dense matter

- However, any given EOS will give a prediction for the maximum mass a neutron star can have before imploding into a black hole.
One way to test EOS is by trying to measure pulsar masses. If one of these is high, then we are able to exclude all equations of state that predict lower maximum possible pulsar masses.

A possible example of this is Terzan 5 I. Its mass could be of the order of 1.9 solar masses. This would exclude most EOS!

This would eliminate most of the “soft” EOS.

Another practical way of excluding EOS would be to find very fast pulsars. The more compact a star, the faster it can, theoretically, spin. The discovery of a sub-millisecond pulsar would eliminate most of the “hard” equations of state.

So far, the fastest known pulsar is PSR B1937+12, the first millisecond pulsar found. It spins 642 times a second. This has excluded some EOS models already.
Thank you for your time!

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